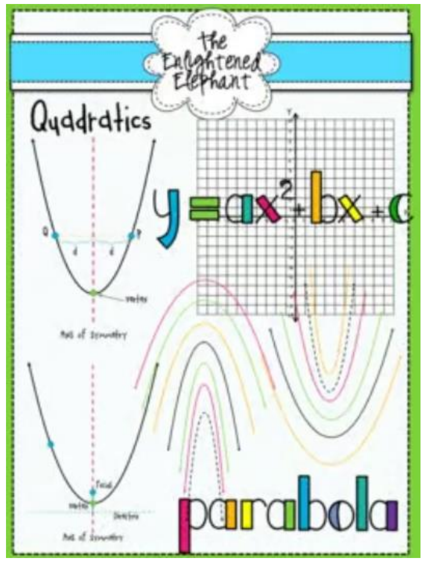
CONIC SECTIONS – THE PARABOLA

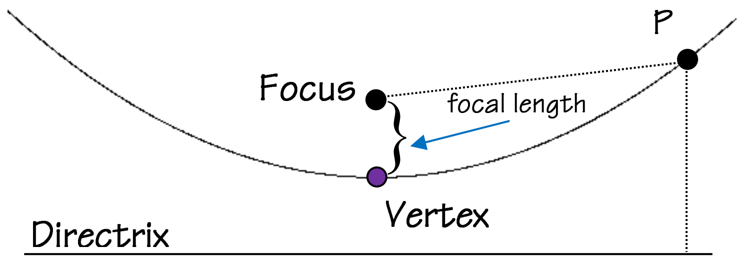


If you've studied the chapter on Parabolas, you will remember that we graphed them by plotting lots of points, and then connected those points to make a smooth curve. That curve then allowed us to estimate the vertex (extreme point) and the intercepts of the graph. In this chapter we get down and dirty and learn where the parabola comes from.

❑ DEFINITION OF THE PARABOLA

Let F be a point in the plane and let L be a line in the plane not containing the point F. Then a ***parabola*** is the set of points satisfying the following condition: *A point is on the parabola if the distance to the point* F *equals the distance to the line* L.

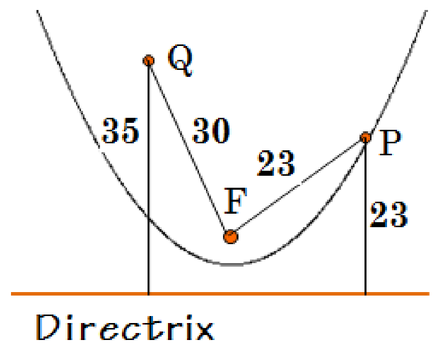
The point F is called the **focus** and the line L is called the **directrix**. The point that is halfway between the focus and the directrix is called the **vertex**. The distance between the vertex and focus (or between the vertex and the directrix) is called the **focal length**.



If point P is on the parabola, then the distance from P to the focus must equal the distance from P to the directrix.

If the focus is above the directrix, we will agree that the focal length is positive, but if the focus is below the directrix, we'll regard the focal length as negative.

For example, suppose that the point P is on the parabola to the right, and assume that the distance between P and the focus, F, of the parabola is 23. Then what is the distance between P and the directrix? It must be **23**, because if P is on the parabola, the distance between P and the focus <u>must</u> equal the distance between P and the directrix.



On the other hand, point Q is <u>not</u> on the parabola because it is 30 units from Q to the focus, whereas it is 35 units from Q to the directrix. Since the distances don't match, Q cannot be on the parabola.

Homework

1. Here's a simpler way to write the definition of the parabola: *A parabola is the set of points in the plane which are equidistant from a given point and a given line.*
   a. What is the given point called?
   b. What is the given line called?

2. a. Sketch a parabola whose focus is below the directrix.

   b. Sketch a parabola whose focus is to the right of the directrix.

   c. Sketch a parabola whose focus is to the left of the directrix.

   d. What might be the reason that in this chapter we won't work with parabolas like those in parts b. and c. of this question?

3. a. If the focal length is positive, the parabola opens ____.

   b. If the focal length is negative, the parabola opens ____.

4. Suppose Q is a point on a parabola, and assume that the distance between Q and the directrix is 197. What is the distance between Q and the focus?

5. Let F be the focus of a parabola, and let P be a point in the plane such that the distance between P and F is 30 and the distance between P and the directrix is 25. Explain why point P is <u>not</u> on the parabola.

6. In each problem, the vertex and focal length of a parabola are given. Find the focus and the directrix.

   a. V(0, 0) $p = 3$ b. V(0, 0) $p = -7$

   c. V(2, 3) $p = 5$ d. V(2, 3) $p = -5$

   e. V(–5, 1) $p = 2$ f. V(–3, –7) $p = -1$

7. In each problem, the focus and the directrix are given. Find the vertex.

   a. F(2, 3) directrix: $y = 1$ b. F(3, 7) directrix: $y = 9$

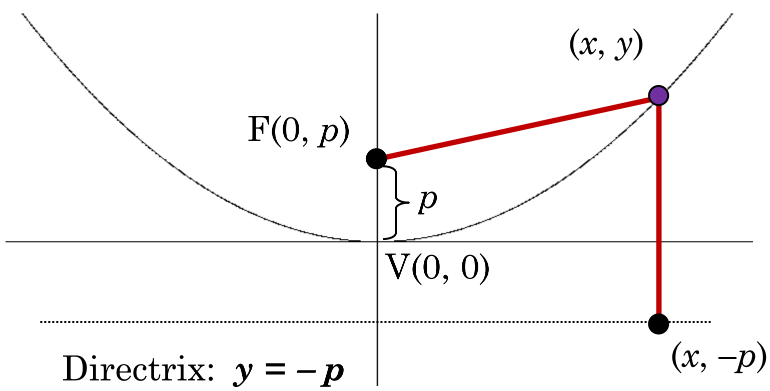
   c. F(–8, –4) directrix $y = 2$ d. F(0, –10) directrix: $y = -30$

❑ THE FUNDAMENTAL PARABOLA FORMULA

THEOREM: Consider the parabola whose vertex is at the origin (0, 0) and which opens up. Let p represent the focal length (note that, in this case, $p > 0$). Then the formula for the parabola is given by

$$\boxed{y = \frac{1}{4p}x^2}$$

PROOF: Consider the following diagram of a parabola with vertex at the origin, opening up, with focal length p. Since p is the distance between the vertex and the focus, the coordinates of the focus are $(0, p)$. Since the directrix is p units down from the origin, its equation is the horizontal line $y = -p$. A generic point (x, y) has been labeled on the parabola — this point, in a sense, represents all the points on the parabola. Also note the point $(x, -p)$ on the directrix. Its first coordinate must be the first coordinate of the point directly above it, so it's x. The second coordinate must be $-p$, since it's on the line $y = -p$.



In order that the point (x, y) be on the parabola, it must satisfy the definition of the parabola: The distance from (x, y) to $(0, p)$

must equal the distance from (x, y) to $(x, -p)$. Recalling the Distance Formula, we can write

$$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{(x-x)^2+[y-(-p)]^2}$$

$$\Rightarrow \quad \sqrt{x^2+y^2-2py+p^2} = \sqrt{0^2+(y+p)^2}$$

$$\Rightarrow \quad \sqrt{x^2+y^2-2py+p^2} = \sqrt{y^2+2py+p^2}$$

$$\Rightarrow \quad x^2+y^2-2py+p^2 = y^2+2py+p^2$$

$$\Rightarrow \quad x^2+\cancel{y^2}-2py+\cancel{p^2} = \cancel{y^2}+2py+\cancel{p^2}$$

$$\Rightarrow \quad x^2-4py = 0$$

$$\Rightarrow \quad -4py = -x^2$$

$$\Rightarrow \quad y = \frac{-x^2}{-4p}$$

$$\Rightarrow \quad \boxed{y = \frac{1}{4p}x^2}$$

<u>Q.E.D.</u>

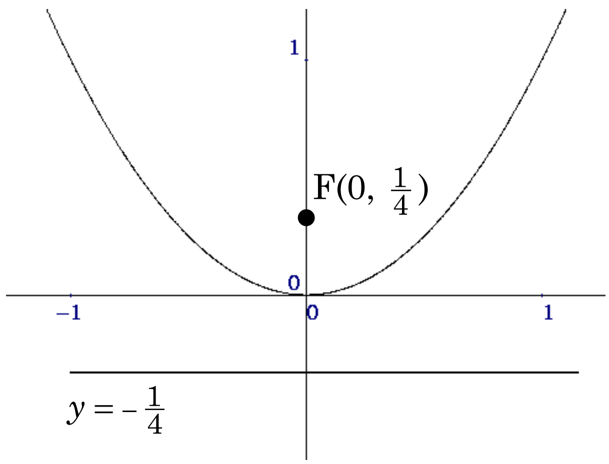
<u>EXAMPLE 1:</u> **Find the focal length of the classic parabola $y = x^2$. Now state the vertex, the focus, and directrix of the parabola.**

<u>Solution:</u> In the formula $y = \frac{1}{4p}x^2$, the coefficient of the x^2 term is $\frac{1}{4p}$. For the classic parabola in this example, the coefficient of the x^2 term is 1. Hence, setting the coefficients equal to each other will yield us the value of p, the focal length.

$$\frac{1}{4p}=1 \quad \Rightarrow \quad 1=4p \quad \Rightarrow \quad \boldsymbol{p=\frac{1}{4}}$$

Since $p > 0$, the parabola opens up. Because $p = \frac{1}{4}$, and the vertex is at the origin, the focus is $\frac{1}{4}$ unit up from (0, 0), which is **(0, $\frac{1}{4}$)**.

The directrix is therefore $\frac{1}{4}$ unit down from (0, 0), and so the equation of the directrix is the horizontal line $\boldsymbol{y = -\frac{1}{4}}$.



<u>EXAMPLE 2:</u> **Find the vertex, the focus, and the directrix of the parabola $\boldsymbol{y = \frac{1}{8}x^2}$.**

<u>Solution:</u> Comparing $y = \frac{1}{8}x^2$ with $y = \frac{1}{4p}x^2$, we can write

$$\frac{1}{4p} = \frac{1}{8} \quad \Rightarrow \quad 8 = 4p \quad \Rightarrow \quad p = 2$$

So the focus is 2 units above the vertex (0, 0): **F(0, 2)**.

The directrix is 2 units below the vertex (0, 0): $\boldsymbol{y = -2}$.

<u>EXAMPLE 3:</u> **Find the vertex, the focus, and the directrix of the parabola $\boldsymbol{y = -2x^2}$.**

<u>Solution:</u> Now that we're getting the hang of it, we see that

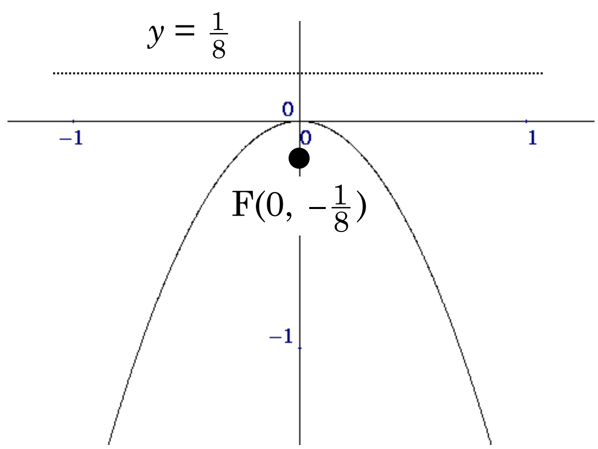
1) The vertex is **(0, 0)**.

2) $\frac{1}{4p} = -2 \;\Rightarrow\; 1 = -8p \;\Rightarrow\; p = -\frac{1}{8}$

Note that $p < 0$; thus the parabola opens down.

3) The focus is $\frac{1}{8}$ unit below (0, 0); therefore, $\mathbf{F(0,\ -\frac{1}{8})}$.

4) The directrix is $\frac{1}{8}$ unit above (0, 0); thus, $\boldsymbol{y = \frac{1}{8}}$.



Homework

8. Find the vertex, the focus, and the directrix of each parabola:

a. $y = -x^2$ b. $y = \frac{1}{12}x^2$ c. $y = -\frac{1}{16}x^2$

d. $y = 3x^2$ e. $y = -4x^2$ f. $y = 2x + 3$

❑ *Shifting the Parabola*

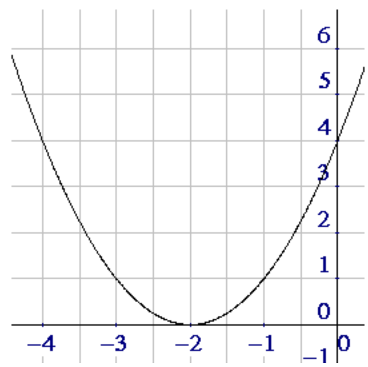
As we did with square root graphs, absolute value graphs, and cube root graphs, we now analyze what happens when the parabola formula we've been using has extra numbers hanging around.

EXAMPLE 4: **Graph the two parabolas** $\boldsymbol{y = (x+2)^2}$ **and** $\boldsymbol{y = x^2 - 3}$**.**

Solution: We'll graph these two functions by plotting some points, kind of pretending that we don't already know that they're parabolas.

$$y = (x+2)^2$$

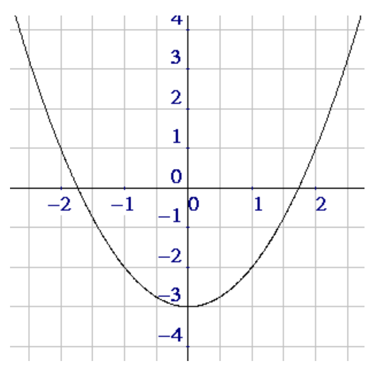
x	y
−4	4
−3	1
−2	0
−1	1
0	4



We notice that this parabola is the parabola $y = x^2$ shifted 2 units to the left. This is the same concept we've seen with other functions. Now for the other parabola:

$$y = x^2 - 3$$

x	y
−2	1
−1	−2
0	−3
1	−2
2	1

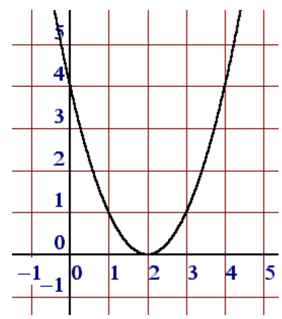


Again we see a familiar shift, in this case 3 units down. We reach the same conclusion we have in the past. When there's a number next to the x (being added or subtracted), there's a horizontal shift in the direction opposite of the sign of the number. When there's a number hanging off the end of the function, we get a vertical shift in the same direction as the sign of the number.

EXAMPLE 5: **Find the vertex of each parabola:**

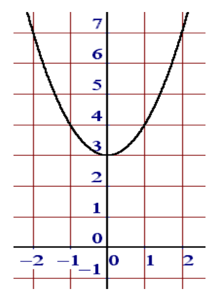
A. $y = (x - 2)^2$

The vertex of the standard parabola $y = x^2$ is (0, 0). Since there's a -2 next to the x, the vertex is shifted 2 units to the right. Therefore, the vertex of $y = (x - 2)^2$ is **(2, 0)**.

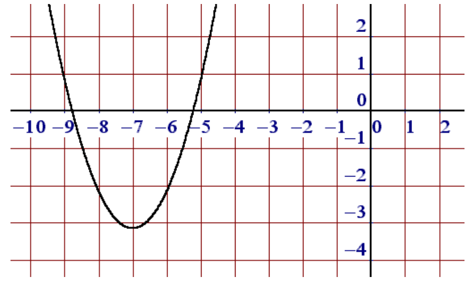


B. $y = x^2 + 3$

The vertex of the standard parabola $y = x^2$ is (0, 0). Because there's a 3 hanging off the end of the formula, the vertex is shifted 3 units up. Thus, the vertex of $y = x^2 + 3$ is **(0, 3).**

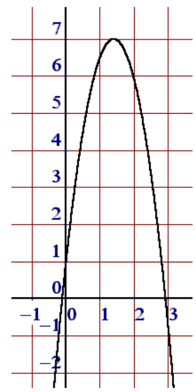


C. $y = (x + 7)^2 - \pi$



The 7 yields a shift to the left of 7 units. The $-\pi$ produces a shift down of π units. And so the graph has its vertex at the point **$(-7, -\pi)$**.

D. $y = -3(x - \sqrt{2})^2 + 7$



The -3 simply makes the parabola open down and appear skinnier than $y = x^2$, but it does not affect the vertex. The usual vertex of (0, 0) is shifted $\sqrt{2}$ units to the right and 7 units up. Therefore, the new vertex is at **$(\sqrt{2}, 7)$.**

<u>EXAMPLE 6:</u> **Analyze the parabola $y = \frac{1}{32}(x-3)^2 + 2$. Include the domain, the y-intercept, the x-intercept(s), the vertex, the focus, the directrix, and the graph.**

<u>Solution:</u> This is one tall order, but if we proceed one step at a time, and remember everything we've learned so far, we can solve this problem.

Domain: We've seen before that the domain of this function is all real numbers; after all, there's no division to worry about, and no square roots to concern us. Thus, x can be any number at all, and so **the domain is $\mathbb{R}$**.

y-intercept: As always, y-intercepts are found by setting $x = 0$ and solving for y. We get

$$y = \frac{1}{32}(\mathbf{0}-3)^2+2 = \frac{1}{32}(9)+2 = \frac{9}{32}+\frac{64}{32} = \frac{73}{32}$$

and so **the *y*-intercept is $\mathbf{(0, \frac{73}{32})}$**.

x-intercept(s): Setting $y = 0$ gives the equation

$$0 = \frac{1}{32}(x-3)^2+2 \Rightarrow -2 = \frac{1}{32}(x-3)^2 \Rightarrow -64 = (x-3)^2.$$

But this last equation has no solution, since the square of a quantity can never be negative. Since we were seeking x-intercepts, and we ran into an unsolvable equation, we conclude that our parabola has **no *x*-intercepts**.

Vertex: By looking at the equation, we see a horizontal shift of 3 units to the right and a vertical shift of 2 units up. Therefore, **the vertex is (3, 2)**.

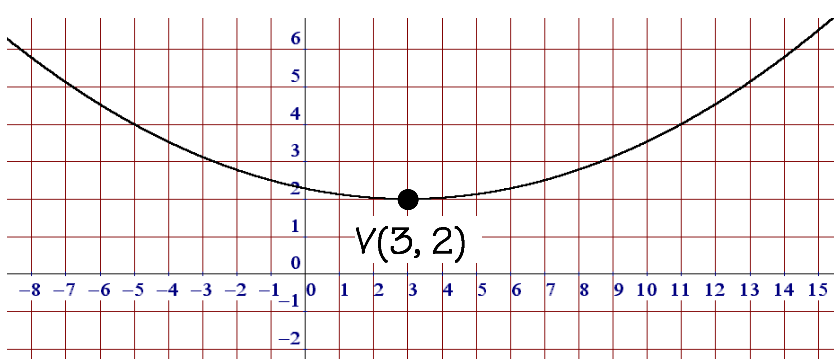
Focus: To find the focus we need the focal length. We know that the coefficient $\frac{1}{32}$ must equal $\frac{1}{4p}$. Thus,

$$\frac{1}{4p} = \frac{1}{32} \Rightarrow 4p = 32 \Rightarrow p = 8$$

Since $p > 0$, we know that the parabola opens up. And the focus must therefore be 8 units above the vertex. Since the vertex is at (3, 2), **the focus is at (3, 10)**.

Directrix: In a parabola which opens up, the directrix is the horizontal line p units below the vertex. So, the line which is 8 units below the vertex (3, 2) is ***y* = –6**.

Graph: Plotting the vertex, then the y-intercept, approximately (0, 2.3), and a few other points yields the following graph:



EXAMPLE 7: **Determine the vertex, the focus and the directrix of the parabola $y = -4(x + 1)^2 - 3$.**

Solution: Since there's a constant of +1 to the right of the x and a –3 hanging off the end of the function, the vertex is **(–1, –3)**.

To locate the focus, we need the focal length:

$$\frac{1}{4p} = -4 \quad \Rightarrow \quad 1 = -16p \quad \Rightarrow \quad p = -\frac{1}{16}$$

Since $p < 0$, we know that the parabola opens down — and the focus must lie $\frac{1}{16}$ of a unit below the vertex. Since the vertex is (–1, –3), the focus is $\mathbf{(-1, -3\frac{1}{16})}$.

The directrix is $\frac{1}{16}$ of a unit above the vertex. The directrix is therefore the horizontal line $\boldsymbol{y = -2\frac{15}{16}}$.

Homework

9. Find the vertex, the focus, and the directrix of each parabola:

a. $y = 4x^2 + 5$ b. $y = -\frac{1}{4}(x+2)^2$

c. $y = -2(x - 1)^2 - 3$ d. $y = \frac{1}{2}(x-5)^2 + 8$

e. $y = -\frac{1}{12}(x+3)^2 - 10$ f. $y = \frac{1}{20}(x-3)^2 + 8$

❑ *NEW FORM OF THE PARABOLA*

Consider the parabola $y = 2x^2 + 12x + 13$. We know it's a parabola because one variable is squared and the other isn't. But it doesn't have any parentheses in it — that is, it doesn't look anything like the parabolas in the previous section. So how can we find the vertex, focus, and directrix of the parabola in this form?

<u>EXAMPLE 8:</u> **Determine the vertex, the focus and the directrix of the parabola $y = 2x^2 + 12x + 13$.**

<u>Solution:</u> To make the given parabola look like the ones in the previous section, we proceed as follows.

Start with the given parabola:

$$y = 2x^2 + 12x + 13$$

Factor 2 out of the $2x^2$ and the $12x$. Don't worry about the 13 yet:

$$y = 2(x^2 + 6x) + 13$$

Complete the Square in the inner parentheses:

1) take half of 6 to get 3
2) square the 3 to get **9**

Add the "magic number" 9 to the inside of the parentheses. This will produce $x^2 + 6x + 9$, a perfect square trinomial which factors into $(x + 3)^2$. However, how do we justify this adding of 9 to the quantity inside the parentheses?

The first issue to consider is that we did not really add 9 to the right side of the equation. Noting that the 9 we added is actually being multiplied by the 2 in front of the parentheses, <u>we actually added **18**</u> to the right side of the equation.

The second issue is exactly how to balance the adding of 18 to the right side of the equation. The quickest way is to also <u>subtract 18</u> from the right side of the equation (since adding and subtracting 18 is really doing nothing — it's just adding 0 to the right side of the equation). Let's do it:

$$y = 2(x^2 + 6x \boxed{+9}) + 13 \boxed{-18}$$

We actually added 18

This balances the adding of 18

Now factor and then do the arithmetic:

$$y = 2(x + 3)^2 - 5$$

It finally looks the way it ought to. We easily see that **the vertex is (–3, –5)**, and the focal length can be calculated as

$$\frac{1}{4p} = 2 \quad \Rightarrow \quad 1 = 8p \quad \Rightarrow \quad \boldsymbol{p = \frac{1}{8}}$$

Since $p > 0$, the parabola opens up, and therefore the focus is $\frac{1}{8}$ of a unit above the vertex, making **the focus** $\boldsymbol{(-3, -\frac{39}{8})}$. The directrix is $\frac{1}{8}$ of a unit below the vertex, and so **the directrix is** $\boldsymbol{y = -\frac{41}{8}}$.

<u>EXAMPLE 9</u>: **Determine the vertex, the focus and the directrix of the parabola** $\boldsymbol{y = -\frac{1}{4}x^2 + \frac{5}{2}x - \frac{1}{4}}$.

<u>Solution</u>: Completing the square will lead the way to a solution.

$$y = -\frac{1}{4}x^2 + \frac{5}{2}x - \frac{1}{4} \qquad \text{(the given parabola)}$$

$$\Rightarrow \quad y = -\frac{1}{4}(x^2 - 10x) - \frac{1}{4} \qquad \text{(factor)}$$

[One way to find the –10 is to divide $\frac{5}{2}$ by $-\frac{1}{4}$]

$$\Rightarrow \quad y = -\frac{1}{4}(x^2 - 10x \boxed{+25}) - \frac{1}{4} \boxed{+\frac{25}{4}} \qquad \text{(complete the square)}$$

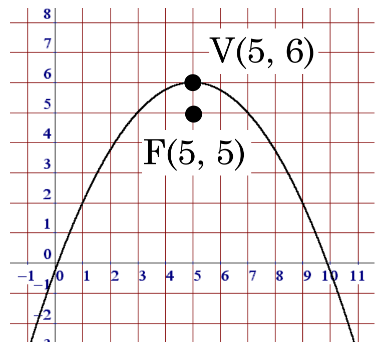
[Note: $-\frac{1}{4}\times 25 = -\frac{25}{4}$; this is balanced by adding $\frac{25}{4}$.]

$\Rightarrow \qquad y = -\frac{1}{4}(x-5)^2 + 6$ (factor & arithmetic)

We now have the parabola in the form we like, from which we deduce that **the vertex is (5, 6)**. The focal length is calculated as:

$\frac{1}{4p} = -\frac{1}{4} \Rightarrow -4p = 4 \Rightarrow \boldsymbol{p = -1}$,

and so the parabola opens down. **The focus is (5, 5)**, 1 unit below the vertex, and **the directrix is $\boldsymbol{y = 7}$**, 1 unit above the vertex.



Homework

10. For each parabola, determine whether the parabola opens up or down, and then compute its focal length.

a. $y = \frac{1}{8}x^2 + \frac{3}{4}x - \frac{31}{8}$ b. $6y = x^2 - 8x + 22$

c. $x^2 + 4x + 4y + 16 = 0$ d. $y = -x^2 + 10x - 21$

11. Find the vertex, the focus, and the directrix of each parabola:

a. $5y - 54 = x^2 + 14x$ b. $x^2 + 2x + 3y + 16 = 0$

c. $y - x^2 + 8x - 9 = 0$ d. $y = -\frac{1}{9}x^2 + \frac{20}{9}x - \frac{73}{9}$

12. Find the vertex, the focus, and the directrix of each parabola:

a. $(x-3)^2 = 20(y+2)$ b. $(x+5)^2 = -8(y-1)$

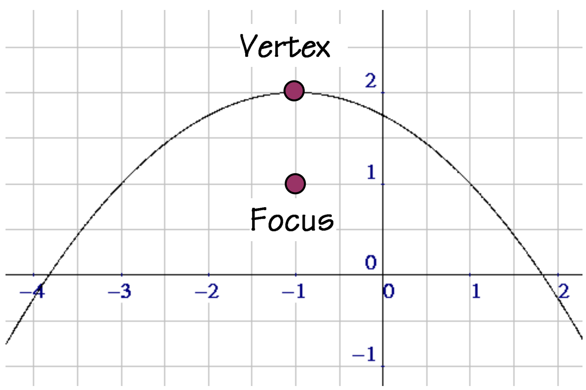
c. $x^2 + 2x + 4y - 15 = 0$ d. $y = 2x^2 - 8x + 7$

13. Graph the parabola $y = \dfrac{x^2 - 8x - 8}{12}$. Label the vertex and the focus. Also give the y-intercept and use the Quadratic Formula to estimate the x-intercepts.

14. Find the equation of the parabola given that

a. its vertex is $(2, -7)$ and $p = 5$.
b. its focus is $(-3, 4)$ and $p = -3$.
c. its vertex is $(-3, 1)$ and its focus is $(-3, 7)$.
d. its vertex is $(4, -1)$ and its focus is $(4, -2)$.

15. Find the equation of the following parabola, and state the equation of the directrix:



Review Problems

16. Give the geometric definition of the parabola.

17. Suppose R is a point on a parabola, and assume that the distance between R and the focus is 250. What is the distance between R and the directrix?

18. The vertex of a parabola is V(3, –10) and the focal length is $p = -4$. Calculate the focus and the directrix of the parabola.

19. Find the focal length of $y = -\frac{7}{8}x^2$.

20. Consider the parabola $y = -9(x+7)^2 - 20$.
    a. Does it open up or down?
    b. Is it skinnier or wider than the $y = x^2$ parabola?
    c. Find the vertex.

21. Give a full analysis of the parabola $y = \frac{1}{24}(x-3)^2 - 5$.

22. Find the vertex and the focal length of the parabola $y = -3x^2 - 24x + 8$.

23. a. Let $y = -\frac{1}{12}x^2 + \frac{1}{3}x - \frac{13}{3}$. Find the domain and all the intercepts.
    b. Find the vertex, the focus, and the directrix.

24. Find the equation of the parabola with vertex (–2, 1) and focus (–2, 7).

25. Derive the parabola formula $y = \frac{1}{4p}x^2$.

26. True/False:

a. A parabola is the set of all points in the plane such that the sum of the distances to a point and a line is a constant.

b. A parabola is the set of all points in the plane that are equidistant from a point and a line.

c. All parabolas are functions.

d. If the vertex of a parabola is V(2, 3) and the focal length is $p = 5$, then the focus is F(2, 8).

e. The focus is always midway between the vertex and the directrix.

f. In the standard parabola formula $y = \frac{1}{4p}x^2$, the p represents the focal length.

g. The derivation of the formula above is based upon the Distance Formula.

h. The focal length of the parabola $y = x^2$ is 1/2.

i. The vertex of the parabola $y = x^2$ is the origin.

j. The vertex of the parabola $y = 2(x - 3)^2 + 4$ is V(–3, 4).

k. The parabola above has no x-intercepts.

l. The vertex of the parabola $y = 2x^2 + 12x + 13$ is V(–3, –5).

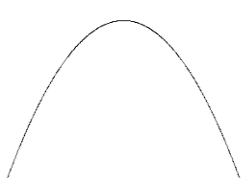
m. The parabola $x = -y^2$ opens down.

n. The graph of $y^2 = 2(x - 3)^2 + 7$ is a parabola.

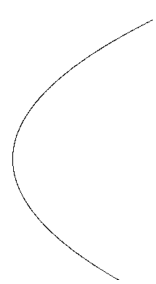
Solutions

1. a. Focus b. Directrix

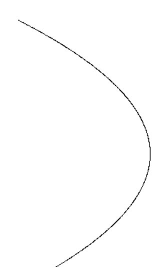
2. a.



b.



c.



d. Hint: Look at the chapter title

3. a. Up b. Down

4. 197

5. In order for a point to be on the parabola, the distance between that point and the focus must equal the distance from that point to the directrix. Since the given distances don't match, the point cannot possibly be on the parabola.

6. a. F(0, 3); $y = -3$ b. F(0, –7); $y = 7$ c. F(2, 8); $y = -2$
d. F(2, –2); $y = 8$ e. F(–5, 3); $y = -1$ f. F(–3, –8); $y = -6$

7. a. V(2, 2) b. V(3, 8) c. V(–8, –1) d. V(0, –20)

8.

a.	V(0, 0)	F$\left(0, -\frac{1}{4}\right)$	$y = \frac{1}{4}$
b.	V(0, 0)	F(0, 3)	$y = -3$
c.	V(0, 0)	F(0, –4)	$y = 4$
d.	V(0, 0)	F$\left(0, \frac{1}{12}\right)$	$y = -\frac{1}{12}$
e.	V(0, 0)	F$\left(0, -\frac{1}{16}\right)$	$y = \frac{1}{16}$
f.	A line has no vertex, focus, or directrix		

9.

a.	V(0, 5)	F$\left(0, 5\frac{1}{16}\right)$	$y = 4\frac{15}{16}$
b.	V(–2, 0)	F(–2, –1)	$y = 1$
c.	V(1, –3)	F$\left(1, -3\frac{1}{8}\right)$	$y = -2\frac{7}{8}$
d.	V(5, 8)	F$\left(5, 8\frac{1}{2}\right)$	$y = 7\frac{1}{2}$
e.	V(–3, –10)	F(–3, –13)	$y = -7$
f.	V(3, 8)	F(3, 13)	$y = 3$

10. a. Up; 2 b. Up; $\frac{3}{2}$ c. Down; -1 d. Down; $-\frac{1}{4}$

11. a. V(–7, 1) F(–7, $\frac{9}{4}$) $y = -\frac{1}{4}$

b. V(–1, –5) F(–1, $-\frac{23}{4}$) $y = -\frac{17}{4}$

c. V(4, –7) F(4, $-\frac{27}{4}$) $y = -\frac{29}{4}$

d. V(10, 3) F(10, $\frac{3}{4}$) $y = \frac{21}{4}$

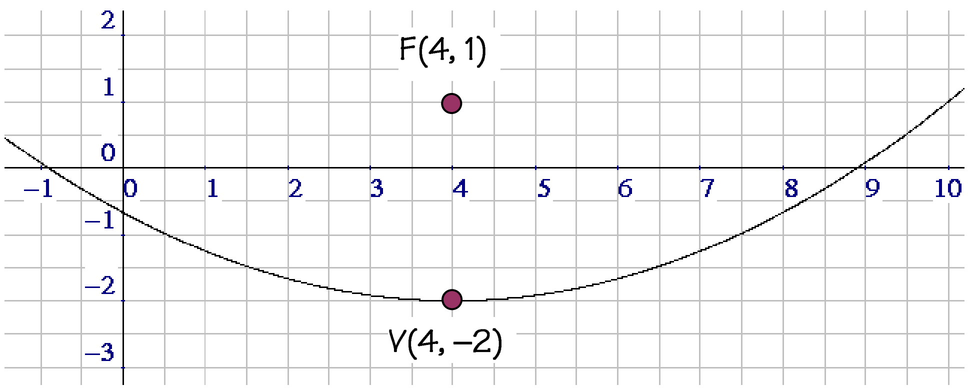
12. a. V(3, –2) F(3, 3) $y = -7$

b. V(–5, 1) F(–5, –1) $y = 3$

c. V(–1, 4) F(–1, 3) $y = 5$

d. V(2, –1) F(2, $-\frac{7}{8}$) $y = -\frac{9}{8}$

13.



The y-intercept is (0, $-\frac{2}{3}$).

The x-intercepts are approximately at (8.9, 0) and (–0.9, 0).

14. a. $y = \frac{1}{20}(x-2)^2 - 7$ b. $y = -\frac{1}{12}(x+3)^2 + 7$

c. $y = \frac{1}{24}(x+3)^2 + 1$ d. $y = -\frac{1}{4}(x-4)^2 - 1$

15. $y = -\frac{1}{4}(x+1)^2 + 2$; The directrix is $y = 3$.

16. The set of all points in the plane which are equidistant from a given point and a given line.

17. 250 **18.** F(3, –14) directrix: $y = -6$ **19.** $p = -\frac{2}{7}$

20. a. Down b. Skinnier c. V(–7, –20)

21. y-intercept: setting $x = 0$ gives

$$y = \frac{1}{24}(-3)^2 - 5 = \frac{9}{24} - 5 = \frac{3}{8} - 5 = -\frac{37}{8}.$$

The y-intercept is $\left(0, -\frac{37}{8}\right)$.

x-intercepts: setting $y = 0$ gives

$$\frac{1}{24}(x-3)^2 - 5 = 0 \Rightarrow \frac{1}{24}(x-3)^2 = 5 \Rightarrow (x-3)^2 = 120$$

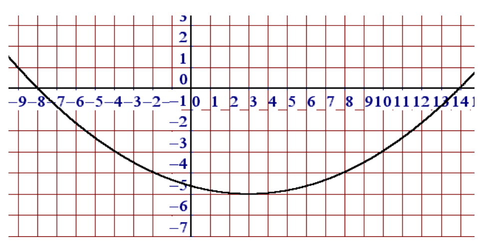
$$\Rightarrow x - 3 = \pm\sqrt{120} \Rightarrow x = 3 \pm 2\sqrt{30}.$$

The x-intercepts are $(3 \pm 2\sqrt{30}, 0)$.

The parabola opens up. The vertex is V(3, –5).

The focal length is $p = 6$. The focus is F(3, 1).

The directrix is $y = -11$.



22. $y = -3x^2 - 24x + 8 = -3(x^2 + 8x) + 8 = -3(x^2 + 8x + 16) + 8 + 48$

$= -3(x + 4)^2 + 56$

The vertex is (–4, 56) and the focal length is $p = -\frac{1}{12}$.

23. a. Domain = $\mathbb{R}$; no x-intercepts; y-int: $(0, -\frac{13}{3})$

b. V(2, –4); F(2, –7); directrix: $y = -1$

24. $y = \frac{1}{24}(x+2)^2 + 1$

25. It's in the book.

26. a. F b. T c. F d. T e. F f. T g. T
h. F i. T j. F k. T l. T m. F n. F

Consider the postage stamp.
It secures success through its ability
to stick to one thing till it gets there.

Josh Billings